Lesson 4: Solutions of a Linear Equation

☐ I can transform equations into a simpler form using the distributive property.
☐ I can describe when a linear equation has one solution, no solution, or infinitely many solutions.

Lesson Notes

The distributive property is used to expand expressions. For example, the expression $2(3x - 10)$ is rewritten as $6x - 20$ after the distributive property is applied.

The distributive property is used to simplify expressions. For example, the expression $7x + 11x$ is rewritten as $(7 + 11)x$ and $18x$ after the distributive property is applied.

The distributive property is applied only to terms within a group:

$$4(3x + 5) - 2 = 12x + 20 - 2.$$

Notice that the term -2 is not part of the group and, therefore, not multiplied by 4.
When an equation is transformed into an untrue sentence, such as $5 \neq 11$, we say the equation has *no solution*.

Exercises

Find the value of x that makes the equation true.

1. $17 - 5(2x - 9) = -(-6x + 10) + 4$

2. $-(x - 7) + \frac{5}{3} = 2(x + 9)$

3. $\frac{4}{9} + 4(x - 1) = \frac{28}{9} - (x - 7x) + 1$

4. $5(3x + 4) - 2x = 7x - 3(-2x + 11)$

5. $7x - (3x + 5) - 8 = \frac{1}{2}(8x + 20) - 7x + 5$

6. Write at least three equations that have no solution.

Lesson Notes

There are three classifications of solutions to linear equations: one solution (unique solution), no solution, or infinitely many solutions.

Equations with no solution will, after being simplified, have coefficients of x that are the same on both sides of the equal sign and constants that are different. For example, $x + b = x + c$, where b and c are constants that are not equal. A numeric example is $8x + 5 = 8x - 3$.

Equations with infinitely many solutions will, after being simplified, have coefficients of x and constants that are the same on both sides of the equal sign. For example, $x + a = x + a$, where a is a constant. A numeric example is $6x + 1 = 1 + 6x$.

Solve each of the following equations for x.

7. $7x - 3 = 5x + 5$

8. $7x - 3 = 7x + 5$

9. $7x - 3 = -3 + 7x$

Give a brief explanation as to what kind of solution(s) you expect the following linear equations to have. Transform the equations into a simpler form if necessary.

10. $11x - 2x + 15 = 8 + 7 + 9x$

11. $3(x - 14) + 1 = -4x + 5$

12. $-3x + 32 - 7x = -2(5x + 10)$

13. $\frac{1}{2}(8x + 26) = 13 + 4x$

14. Write two equations that have no solutions.

15. Write two equations that have one unique solution each.

16. Write two equations that have infinitely many solutions.